## 7.3.1 Fuzzy Set Operations

The generalization of operations on classical sets to operations on fuzzy sets is not unique. The fuzzy set operations being discussed in this section are termed standard fuzzy set operations. These are the operations widely used in engineering applications. Let $A$ and $B$ be fuzzy sets in the universe of discourse $U$. For a given element $x$ on the universe, the following function theoretic operations of union, intersection and complement are defined for fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ on $U$.

### 7.3.1.1 Union

The union of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$, denoted by $\underset{\sim}{A} \cup \underset{\sim}{B}$, is defined as

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \max[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)] = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x) \quad \text{for all } x \in U$$

where $\vee$ indicates max operation. The Venn diagram for union operation of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is shown in Figure 7-10.

### 7.3.1.2 Intersection

The intersection of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$, denoted by $\underset{\sim}{A} \cap \underset{\sim}{B}$, is defined by

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)] = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x) \quad \text{for all } x \in U$$

where $\wedge$ indicates min operator. The Venn diagram for intersection operation of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is shown in Figure 7-11.

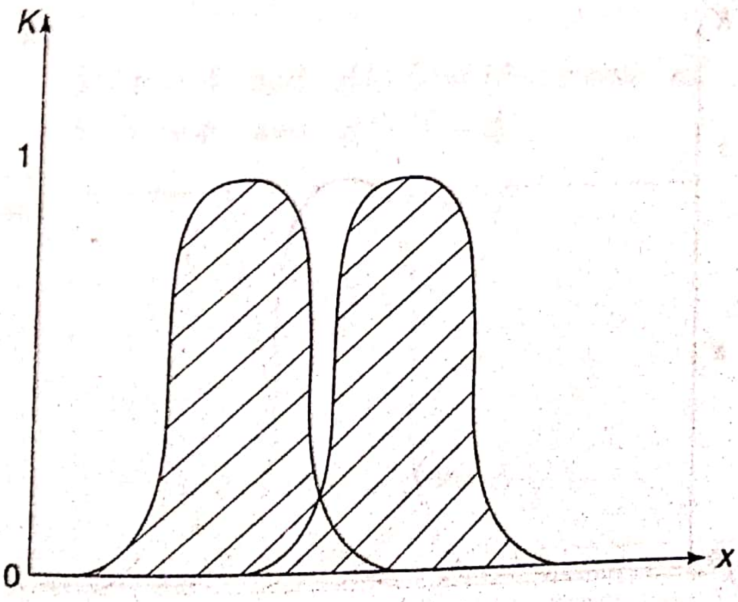


**Figure 7-10** Union of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

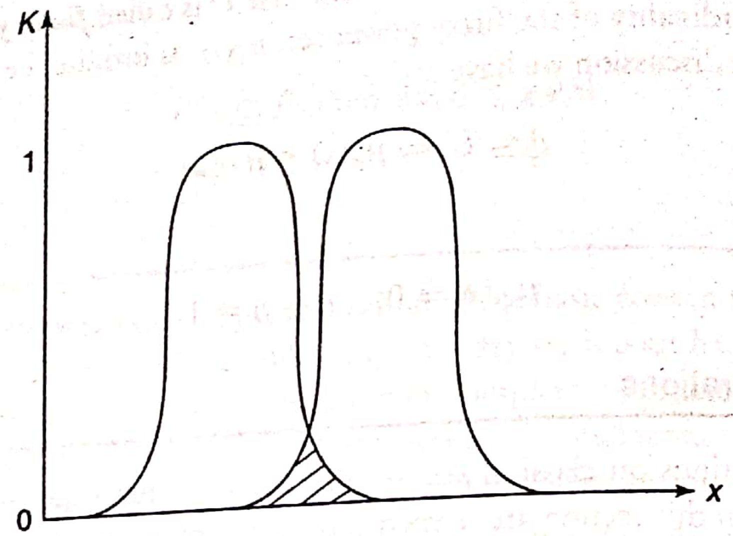


**Figure 7-11** Intersection of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

### *7.3.1.3 Complement*

When $\mu_{\underset{\sim}{A}}(x) \in [0, 1]$, the complement of $\underset{\sim}{A}$, denoted as $\overline{\underset{\sim}{A}}$ is defined by

$$\mu_{\overline{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x) \quad \text{for all } x \in U$$

The Venn diagram for complement operation of fuzzy set $\underset{\sim}{A}$ is shown in Figure 7-12.

### *7.3.1.4 More Operations on Fuzzy Sets*

1. *Algebraic sum:* The algebraic sum $(\underset{\sim}{A} + \underset{\sim}{B})$ of fuzzy sets, fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

2. *Algebraic product:* The algebraic product $(\underset{\sim}{A} \cdot \underset{\sim}{B})$ of two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A} \cdot \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

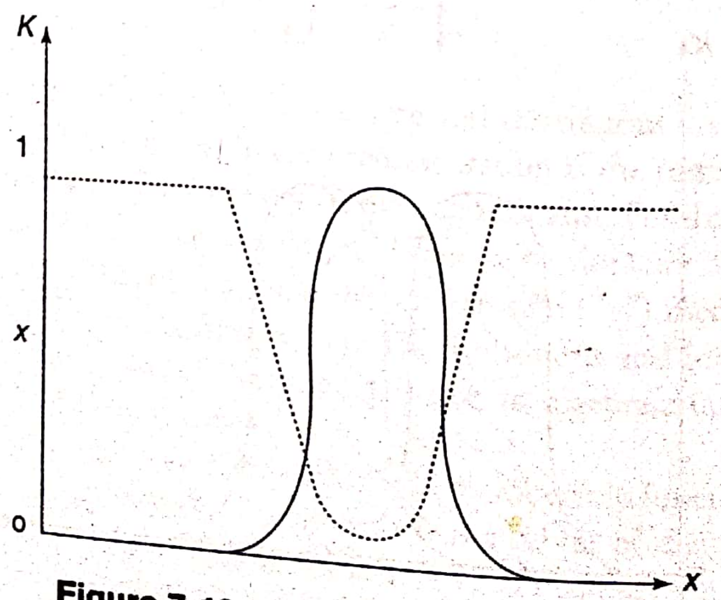


**Figure 7-12** Complement of fuzzy set A

3. *Bounded sum:* The bounded sum $(\underset{\sim}{A} \oplus \underset{\sim}{B})$ of two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A} \oplus \underset{\sim}{B}}(x) = \min\{1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\}$$

4. *Bounded difference:* The bounded difference $(\underset{\sim}{A} \odot \underset{\sim}{B})$ of two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A} \odot \underset{\sim}{B}}(x) = \max\{0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\}$$